## DERIVATIVES

## TAKE THE DERIVATIVES OF THE FOLLOWING:

$$
f(x)=x^{2}+4 x-6
$$

1. $f^{\prime}(x)=2 x+4$
$f^{l}(x)=2(x+2)$
2. $\begin{aligned} & f(x)=x^{3}-5 x^{2}-4 x+11 \\ & f^{l}(x)=3 x^{2}-10 x+4\end{aligned}$

$$
f(x)=\left(x^{6}-x^{4}+2 x\right)
$$

$$
f(x)=\left(x^{12}+x^{7}-x^{3}+2\right)
$$

3. $f^{l}(x)=6 x^{5}-4 x^{3}+2$
4. $f^{l}(x)=12 x^{11}+7 x^{6}-3 x^{2}$

$$
f^{l}(x)=x^{2}\left(12 x^{9}+7 x^{4}-3\right)
$$

5. $f(x)=(x+1)^{2}$
6. $\begin{aligned} & f(x)=(x-5)^{4} \\ & f^{l}(x)=4(x-5)^{3}\end{aligned}$

$$
f(x)=(9 x+5)^{7}
$$

7. $f^{l}(x)=5(4 x+2)^{4} \cdot 4$

$$
f^{l}(x)=20 \cdot 2^{4}(2 x+1)^{4}
$$

$$
f(x)=\left(4 x^{3}+3\right)^{2}
$$

9. $f^{l}(x)=2\left(4 x^{3}+3\right) \cdot 12 x^{2}$

$$
f^{l}(x)=24 x^{2}\left(4 x^{3}+3\right)
$$

11. $f(x)=\left(2 x^{2}+x-1\right)^{-2}$

$$
f^{l}(x)=-2\left(2 x^{2}+x-1\right)^{-3} \cdot(4 x+1)
$$

$$
\begin{aligned}
& f(x)=x^{-3}+2 x^{-4}-3 x^{-1} \\
& f^{l}(x)=-3 x^{-4}-8 x^{-5}+3 x^{-2}
\end{aligned}
$$

13. $f^{l}(x)=x^{-5}\left(-3 x-8+3 x^{3}\right)$

$$
f^{l}(x)=\frac{\left(-3 x-8+3 x^{3}\right)}{x^{5}}
$$

10. $f(x)=\left(5 x^{3}-2 x^{2}+3 x\right)^{3}$

$$
f^{l}(x)=3\left(5 x^{3}-2 x^{2}+3 x\right)^{2} \cdot\left(15 x^{2}-4 x+3\right)
$$

$$
f(x)=\left(4 x^{4}-3 x^{2}+5\right)^{-6}
$$

8. $f^{\prime}(x)=7(9 x+5)^{6} \cdot 9$

$$
f^{l}(x)=63(9 x+5)^{6}
$$

$$
f^{l}(x)=-6\left(4 x^{4}-3 x^{2}+5\right)^{-7} \cdot\left(16 x^{3}-6 x\right)
$$

12. $f^{l}(x)=-12 x\left(4 x^{4}-3 x^{2}+5\right)^{-7} \cdot\left(8 x^{2}-3\right)$

$$
f^{\prime}(x)=\frac{-12 x\left(8 x^{2}-3\right)}{\left(4 x^{4}-3 x^{2}+5\right)^{7}}
$$

$$
f(x)=(2 x+3)(4 x-5)
$$

14. $f(x)=8 x^{2}+2 x-15$
15. 

$$
\begin{aligned}
& f^{l}(x)=16 x+2 \\
& f^{l}(x)=2(8 x+1)
\end{aligned}
$$

$$
\text { 15. } \begin{aligned}
& f(x)=(2 x-7)(4 x+6) \\
& f(x)=8 x^{2}-16 x-42 \\
& f^{l}(x)=16 x-16 \\
& f^{l}(x)=16(x-1)
\end{aligned}
$$

$$
f(x)=\left(5 x^{2}+3 x-2\right)\left(4 x^{4}+2 x^{2}+1\right)
$$

15. 

$f(x)=(4 x-5)^{2}(5 x+3)^{3}$
17. $f^{l}(x)=2(4 x-5) \cdot 4 \cdot(5 x+3)^{3}+3(5 x+3)^{2} \cdot 5 \cdot(4 x-5)^{2}$
$f^{\prime}(x)=(4 x-5)(5 x+3)^{2}[8(5 x+3)+15(4 x-5)]$
$f^{l}(x)=(4 x-5)(5 x+3)^{2}(100 x-51)$

$$
f(x)=(5 x+2)^{3}(7 x-5)^{4}
$$

18. 

$$
f^{l}(x)=3(5 x+2)^{2} \cdot 5 \cdot(7 x-5)^{4}+4(7 x-5)^{3} \cdot 7 \cdot(5 x+2)^{3}
$$

$f^{\prime}(x)=(5 x+2)^{2}(7 x-5)^{3}[15(7 x-5)+28(5 x+2)]$
$f^{l}(x)=(5 x+2)^{2}(7 x-5)^{3}(245 x-19)$
$f(x)=\left(5 x^{2}+5\right)^{5}\left(3 x^{2}-1\right)^{6}$
$f^{l}(x)=5\left(5 x^{2}+5\right)^{4} \cdot 10 x \cdot\left(3 x^{2}-1\right)^{6}+6\left(3 x^{2}-1\right)^{5} \cdot 6 x \cdot\left(5 x^{2}+5\right)^{5}$
19. $f^{l}(x)=2 x\left(5 x^{2}+5\right)^{4}\left(3 x^{2}-1\right)^{5}\left[25\left(3 x^{2}-1\right)+18\left(5 x^{2}+5\right)\right]$
$f^{\prime}(x)=2 x\left(5 x^{2}+5\right)^{4}\left(3 x^{2}-1\right)^{5}\left(165 x^{2}+65\right)$
$f^{\prime}(x)=10 x\left(5 x^{2}+5\right)^{4}\left(3 x^{2}-1\right)^{5}\left(33 x^{2}+13\right)$
$f(x)=\left(3 x^{4}-6\right)^{3}(5 x+4)^{-3}$
$f^{l}(x)=3\left(3 x^{4}-6\right)^{2} \cdot 12 x^{3} \cdot(5 x+4)^{-3}+(-3)(5 x+4)^{-4} \cdot 5 \cdot\left(3 x^{4}-6\right)^{3}$
20. $f^{\prime}(x)=3\left(3 x^{4}-6\right)^{2}(5 x+4)^{-4}\left[12 x^{3}(5 x+4)-5\left(3 x^{4}-6\right)\right]$
$f^{l}(x)=3\left(3 x^{4}-6\right)^{2}(5 x+4)^{-4}\left(45 x^{4}+48 x^{3}+20\right)$
$f^{l}(x)=\frac{3\left(3 x^{4}-6\right)^{2}\left(45 x^{4}+48 x^{3}+20\right)}{(5 x+4)^{4}}$
$f(x)=\left(4 x^{2}+3\right)^{-2}\left(x^{3}+1\right)^{-4}$
$f^{l}(x)=-2\left(4 x^{2}+3\right)^{-3} \cdot 8 x \cdot\left(x^{3}+1\right)^{-4}+(-4)\left(x^{3}+1\right)^{-5} \cdot 3 x^{2} \cdot\left(4 x^{2}+3\right)^{-2}$
21. $f^{l}(x)=-4 x\left(4 x^{2}+3\right)^{-3}\left(x^{3}+1\right)^{-5}\left[4\left(x^{3}+1\right)+3 x\left(4 x^{2}+3\right)\right]$
$f^{l}(x)=-4 x\left(4 x^{2}+3\right)^{-3}\left(x^{3}+1\right)^{-5}\left(16 x^{3}+9 x+4\right)$
$f^{l}(x)=\frac{-4 x\left(16 x^{3}+9 x+4\right)}{\left(4 x^{2}+3\right)^{3}\left(x^{3}+1\right)^{5}}$
$f(x)=\ln (4 x)$

$$
f(x)=\ln \left(5 x^{3}\right)
$$

22. $f^{l}(x)=\frac{1}{4 x} \cdot 4=\frac{1}{x}$
23. $f^{\prime}(x)=\frac{1}{5 x^{3}} \cdot 15 x^{2}=\frac{3}{x}$
$f(x)=(\ln (6 x))^{2}$

$$
f(x)=\ln (6 x)^{2}
$$

24. 

$$
f^{l}(x)=2(\ln (6 x)) \cdot \frac{1}{6 x} \cdot 6=\frac{2}{x}(\ln (6 x))
$$

25. $f^{l}(x)=\frac{1}{(6 x)^{2}} \cdot 2(6 x) \cdot 6=\frac{2}{x}$

$$
f(x)=(2 x+1) \ln x
$$

$$
f(x)=\ln x^{2} \cdot(5 x+2)^{3}
$$

26. $f^{\prime}(x)=2 \ln x+\frac{1}{x}(2 x+1)$
27. $f^{l}(x)=\frac{1}{x^{2}} \cdot 2 x \cdot(5 x+2)^{3}+3(5 x+2)^{2} \cdot 5 \cdot \ln x^{2}$

$$
f^{\prime}(x)=\frac{2}{x} \cdot(5 x+2)^{3}+15(5 x+2)^{2} \cdot \ln x^{2}
$$

$$
\begin{aligned}
& f(x)=\ln \left[(4 x-2)^{2}(5 x+2)^{3}\right] \\
& f^{l}(x)=\frac{1}{\left[(4 x-2)^{2}(5 x+2)^{3}\right]} \cdot\left[2(4 x-2) \cdot(5 x+2)^{3}+3(5 x+2)^{2} \cdot 5 \cdot(4 x-2)^{2}\right]
\end{aligned}
$$

28. 

$$
\begin{aligned}
& f^{\prime}(x)=\frac{1}{\left[(4 x-2)^{2}(5 x+2)^{3}\right]} \cdot(4 x-2)(5 x+2)^{2}[2(5 x+2)+15(4 x-2)] \\
& f^{\prime}(x)=\frac{2(35 x-13)}{\left[(4 x-2)(5 x+2)^{2}\right]}=\frac{2(35 x-13)}{\left[2(2 x-1)(5 x+2)^{2}\right]}=\frac{(35 x-13)}{(2 x-1)(5 x+2)^{2}}
\end{aligned}
$$

$$
f(x)=\ln \left(3 x^{4}+2 x^{2}-5\right)^{3}
$$

29. $f^{l}(x)=\frac{1}{\left(3 x^{4}+2 x^{2}-5\right)^{3}} \cdot 3 \cdot\left(3 x^{4}+2 x^{2}-5\right)^{2} \cdot\left(12 x^{3}+4 x\right)$

$$
f^{\prime}(x)=\frac{12 x\left(3 x^{2}+1\right)}{\left(3 x^{4}+2 x^{2}-5\right)^{2}}
$$

30. $f(x)=4^{3 x}$
$f^{l}(x)=4^{3 x} \cdot \ln 4 \cdot 3$

$$
\text { 31. } \begin{aligned}
f(x) & =5^{\left(3 x^{2}+2 x-5\right)} \\
f^{\prime}(x) & =5^{\left(3 x^{2}+2 x-5\right)} \cdot \ln 5 \cdot(6 x+2) \\
f^{\prime}(x) & =5^{\left(3 x^{2}+2 x-5\right)} \cdot \ln 5 \cdot 2(3 x+1)
\end{aligned}
$$

$$
f(x)=6^{(4 x+5)^{2}(6 x-1)}
$$

32. 

$$
f^{l}(x)=6^{(4 x+5)^{2}(6 x-1)} \cdot \ln 6 \cdot\left[2(4 x+5) \cdot 4 \cdot(6 x-1)+6(4 x+5)^{2}\right]
$$

$$
f^{l}(x)=6^{(4 x+5)^{2}(6 x-1)} \cdot \ln 6 \cdot 2 \cdot(4 x+5)(48 x+26)
$$

$$
f^{l}(x)=6^{(4 x+5)^{2}(6 x-1)} \cdot \ln 6 \cdot 4 \cdot(4 x+5)(24 x+13)
$$

$$
f(x)=4^{\left(5 x^{2}-2 x+1\right)}
$$

33. $f^{l}(x)=4^{\left(5 x^{2}-2 x+1\right)} \cdot \ln 4 \cdot(10 x-2)$
$f^{\prime}(x)=4^{\left(5 x^{2}-2 x+1\right)} \cdot \ln 4 \cdot 2(5 x-1)$
34. $f(x)=e^{(2 x)}$
$f^{\prime}(x)=e^{(2 x)} \cdot 2$

$$
f(x)=5^{(2 x+3)} \ln (2 x+3)
$$

35. $f^{\prime}(x)=5^{(2 x+3)} \cdot \ln 5 \cdot 2 \cdot \ln (2 x+3)+\frac{1}{(2 x+3)} \cdot 2 \cdot 5^{(2 x+3)}$

$$
f^{\prime}(x)=5^{(2 x+3)} \cdot 2\left[\ln 5 \cdot \ln (2 x+3)+\frac{1}{(2 x+3)}\right]
$$

$$
\begin{aligned}
& f(x)=\ln (5 x-3)^{2} e^{(3 x-5)} \\
& f^{l}(x)=\frac{1}{(5 x-3)^{2}} \cdot 2(5 x-3) \cdot 5 \cdot e^{(3 x-5)}+e^{(3 x-5)} \cdot 3 \cdot \ln (5 x-3)^{2}
\end{aligned}
$$

36. 

$$
f^{l}(x)=\frac{1}{(5 x-3)} \cdot 2 \cdot 5 \cdot e^{(3 x-5)}+e^{(3 x-5)} \cdot 3 \cdot \ln (5 x-3)^{2}
$$

$$
f^{l}(x)=e^{(3 x-5)}\left[\frac{10}{(5 x-3)}+3 \cdot \ln (5 x-3)^{2}\right]
$$

$$
f(x)=(3 x+2)^{2}(5 x-1)^{3}(x+6)^{2}
$$

37. $f^{l}(x)=2 \cdot(3 x+2) \cdot 3(5 x-1)^{3}(x+6)^{2}+3(5 x-1)^{2} \cdot 5(3 x+2)^{2}(x+6)^{2}+2(x+6)(3 x+2)^{2}(5 x-1)^{3}$ $f^{l}(x)=(3 x+2)(5 x-1)^{2}(x+6)[6(5 x-1)(x+6)+15(3 x+2)(x+6)+2(5 x-1)(3 x+2)]$

$$
f(x)=\frac{(5 x+2)^{2}}{4 x+1} \Rightarrow f(x)=(5 x+2)^{2}(4 x+1)^{-1}
$$

38. 

$$
f^{l}(x)=2 \cdot(5 x+2) \cdot 5 \cdot(4 x+1)^{-1}+-1(4 x+1)^{-2} \cdot 4 \cdot(5 x+2)^{2}
$$

$$
f^{l}(x)=2(5 x+2)(4 x+1)^{-2}[5(4 x+1)-2(5 x+2)]
$$

$$
f^{\prime}(x)=\frac{2(5 x+2)(10 x+1)}{(4 x+1)^{2}}
$$

$$
f(x)=\frac{\left(5 x^{3}+2 x-5\right)^{4}}{\left(4 x^{2}+3\right)^{2}} \Rightarrow f(x)=\left(5 x^{3}+2 x-5\right)^{4}\left(4 x^{2}+3\right)^{-2}
$$

39. 

$$
f^{l}(x)=4 \cdot\left(5 x^{3}+2 x-5\right)^{3} \cdot\left(15 x^{2}+2\right)\left(4 x^{2}+3\right)^{-2}+-2\left(4 x^{2}+3\right)^{-3} \cdot 8 x \cdot\left(5 x^{3}+2 x-5\right)^{4}
$$

$$
f^{l}(x)=4\left(5 x^{3}+2 x-5\right)^{3}\left(4 x^{2}+3\right)^{-3}\left[\left(15 x^{2}+2\right)\left(4 x^{2}+3\right)-4 x\left(5 x^{3}+2 x-5\right)\right]
$$

$$
f^{\prime}(x)=\frac{4\left(5 x^{3}+2 x-5\right)^{3}\left(40 x^{4}+45 x^{2}+20 x+6\right)}{\left(4 x^{2}+3\right)^{3}}
$$

$$
f(x)=\frac{\ln (5 x-2)^{3}}{5^{5 x-1}}
$$

$$
f^{l}(x)=\frac{\frac{1}{(5 x-2)^{3}} \cdot 3(5 x-2)^{2} \cdot 5 \cdot 5^{5 x-1}-5^{5 x-1} \cdot \ln 5 \cdot 5 \cdot \ln (5 x-2)^{3}}{\left[5^{5 x-1}\right]^{2}}
$$

40. 

$$
\begin{aligned}
& f^{l}(x)=\frac{5^{5 x-1} \cdot 5\left[\frac{3}{(5 x-2)}-\ln 5 \cdot \ln (5 x-2)^{3}\right]}{\left[5^{5 x-1}\right]^{2}}=\frac{5\left[\frac{3-(5 x-2) \ln 5 \cdot \ln (5 x-2)^{3}}{(5 x-2)}\right]}{5^{5 x-1}} \\
& f^{\prime}(x)=\frac{5\left[3-(5 x-2) \ln 5 \cdot \ln (5 x-2)^{3}\right]}{(5 x-2) 5^{5 x-1}}
\end{aligned}
$$

$$
\begin{aligned}
& f(x)=\frac{3^{x^{3}+2 x}}{e^{\ln x}} \\
& f^{\prime}(x)=\frac{3^{x^{3}+2 x} \cdot \ln 3 \cdot\left(2 x^{2}+2\right) e^{\ln x}-e^{\ln x} \cdot \frac{1}{x} \cdot 3^{x^{3}+2 x}}{\left[e^{\ln x}\right]^{2}} \\
& f^{\prime}(x)=\frac{3^{x^{3}+2 x} \cdot e^{\ln x}\left[\ln 3 \cdot\left(2 x^{2}+2\right)-\frac{1}{x}\right]}{\left[e^{\ln x}\right]^{2}}=\frac{3^{x^{3}+2 x}\left[\frac{x \ln 3 \cdot\left(2 x^{2}+2\right)-1}{x}\right]}{e^{\ln x}} \\
& f^{l}(x)=\frac{3^{x^{3}+2 x}\left[x \ln 3 \cdot\left(2 x^{2}+2\right)-1\right]}{x \cdot e^{\ln x}}
\end{aligned}
$$

41. 
